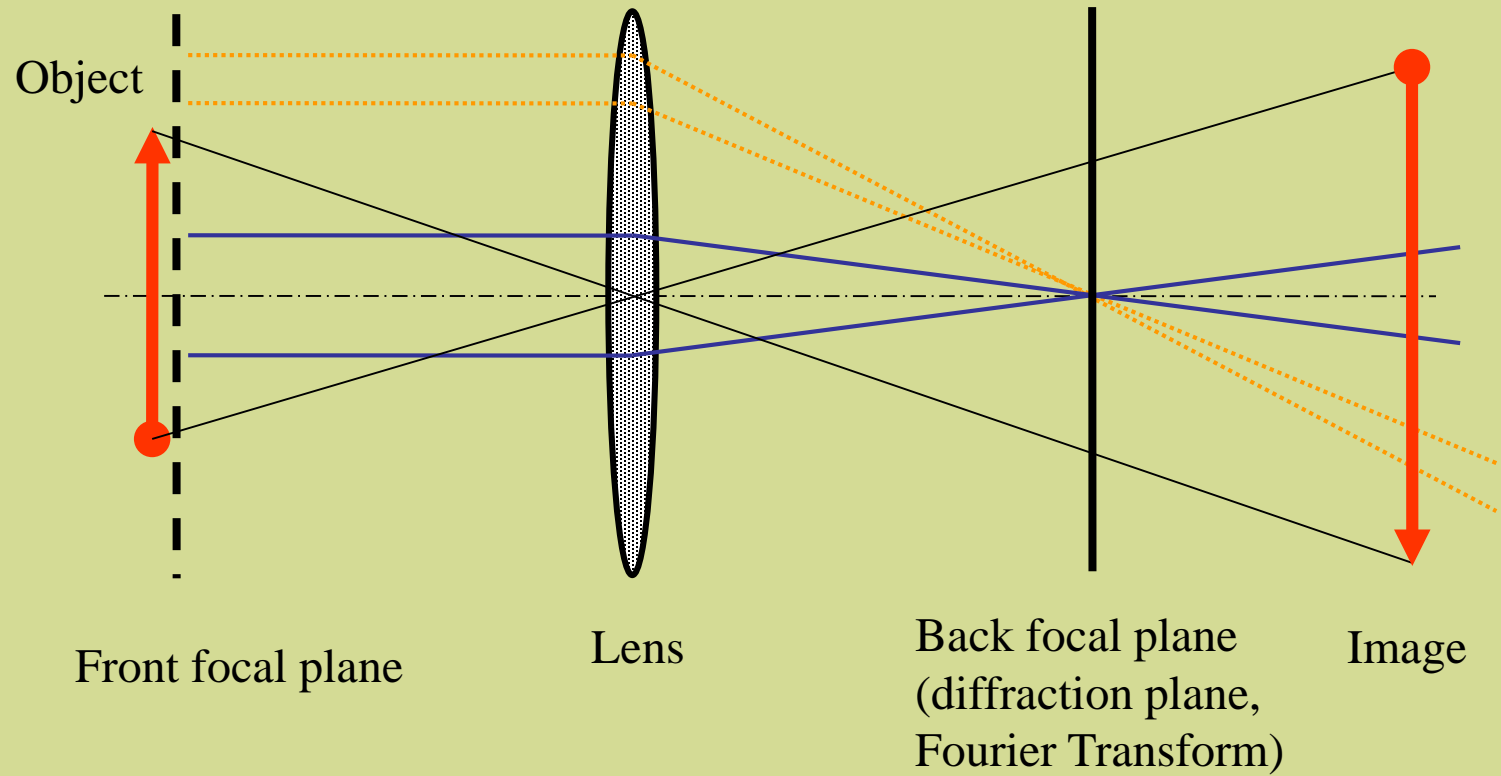


# ***CTF, micrograph evaluation***

***Misha Sherman, UTMB***  
***May 7<sup>th</sup> 2019***

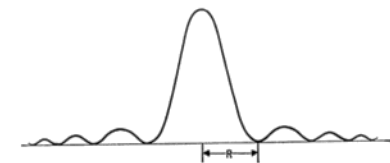
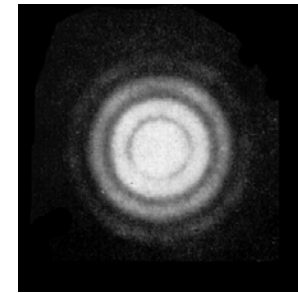
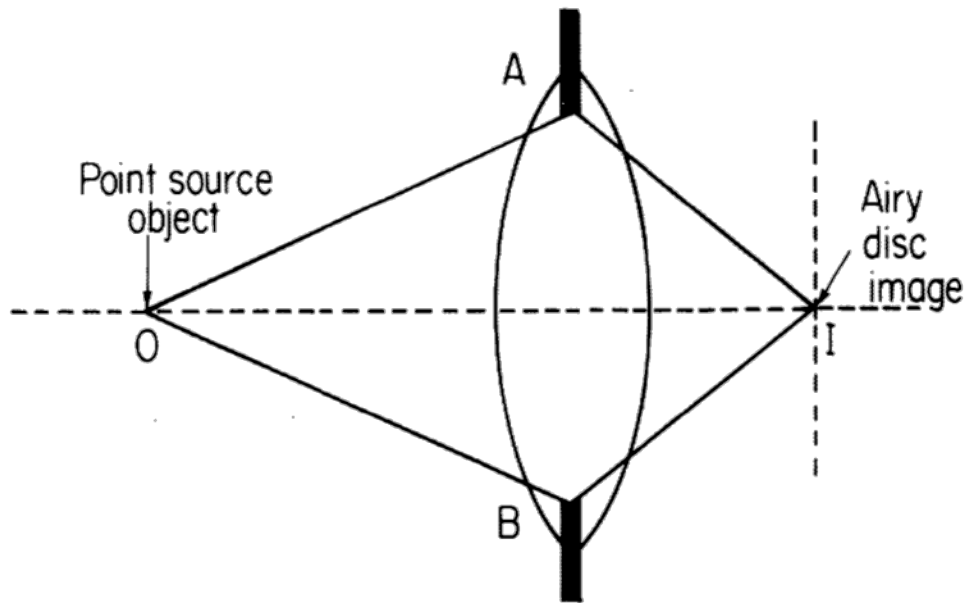
# Lenses



# Optical systems – ideal vs. real

**Ideal lens** – object point  $\rightarrow$  point in the image

**Real lens** – object point  $\rightarrow$  smeared disk in the image



From Meek, 1st ed., Fig. 1.22, p.35  
and Sjostrand, Fig. IV.18, p.115

For axially symmetrical lenses object points  $\rightarrow$  circular disks (Airy disks)

# Phase vs. amplitude contrast

## Phase:

A transparent object varies in refractive index and/or thickness, but does not cause amplitude changes in illumination.

A plane wave of uniform amplitude falls on the specimen and emerges with uniform amplitude  $A_0$  but with phase variations over the plane surface.

$$T(x,y) = A_0 \exp[i\phi(x,y)], \text{ for simplicity } A_0 = 1$$

Assuming that the object is thin and the phase shift  $\phi$  is small ( $\phi \ll 1$ ), the emerged wave can be described as

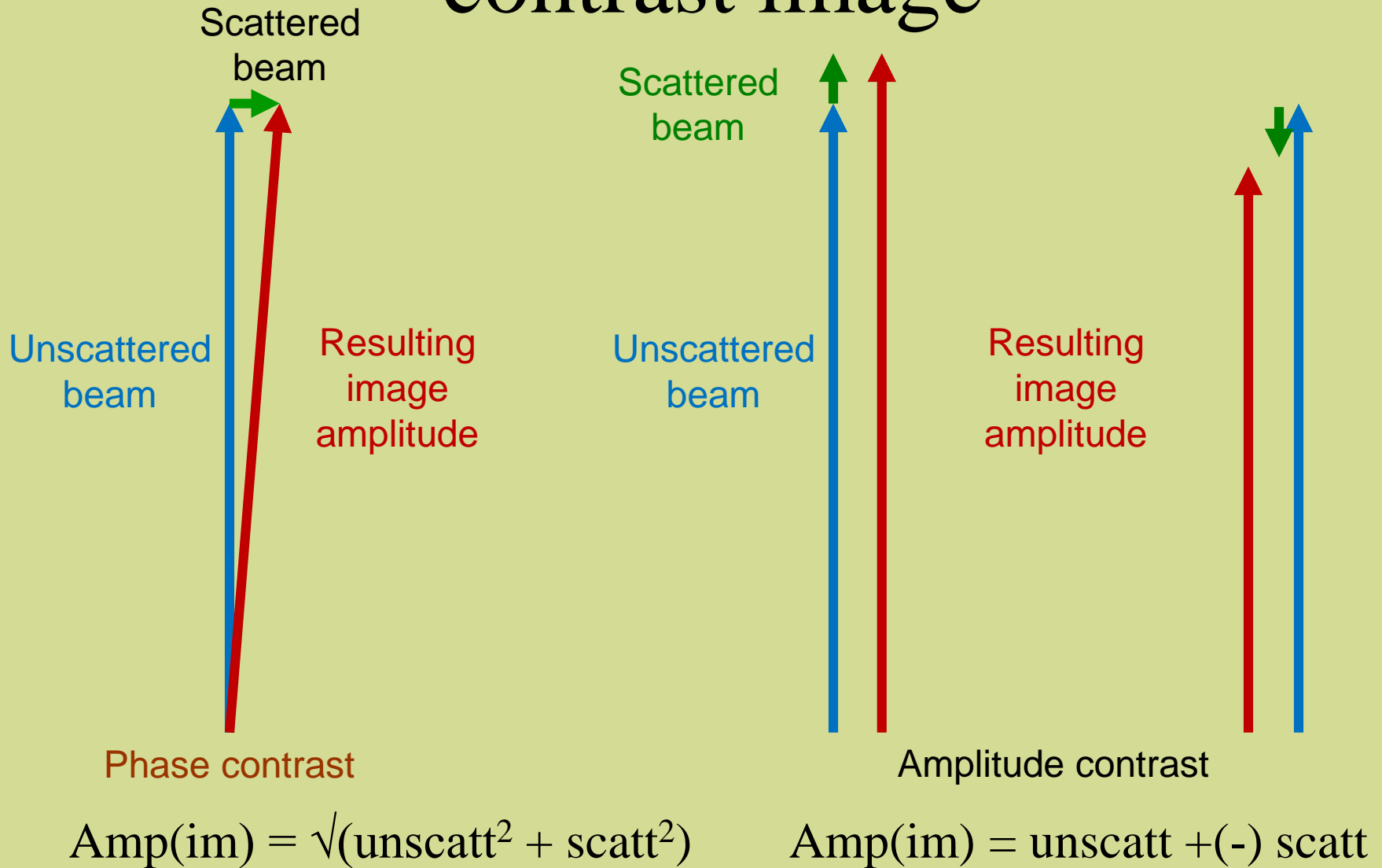
$$\exp [i\phi] \approx 1 + i\phi, \text{ weak phase object;}$$

then  $T(x,y) \approx 1 + i\phi$ ; **Observed intensity**  $T^2(x,y) = 1 - \phi^2 \approx 1$

## Amplitude:

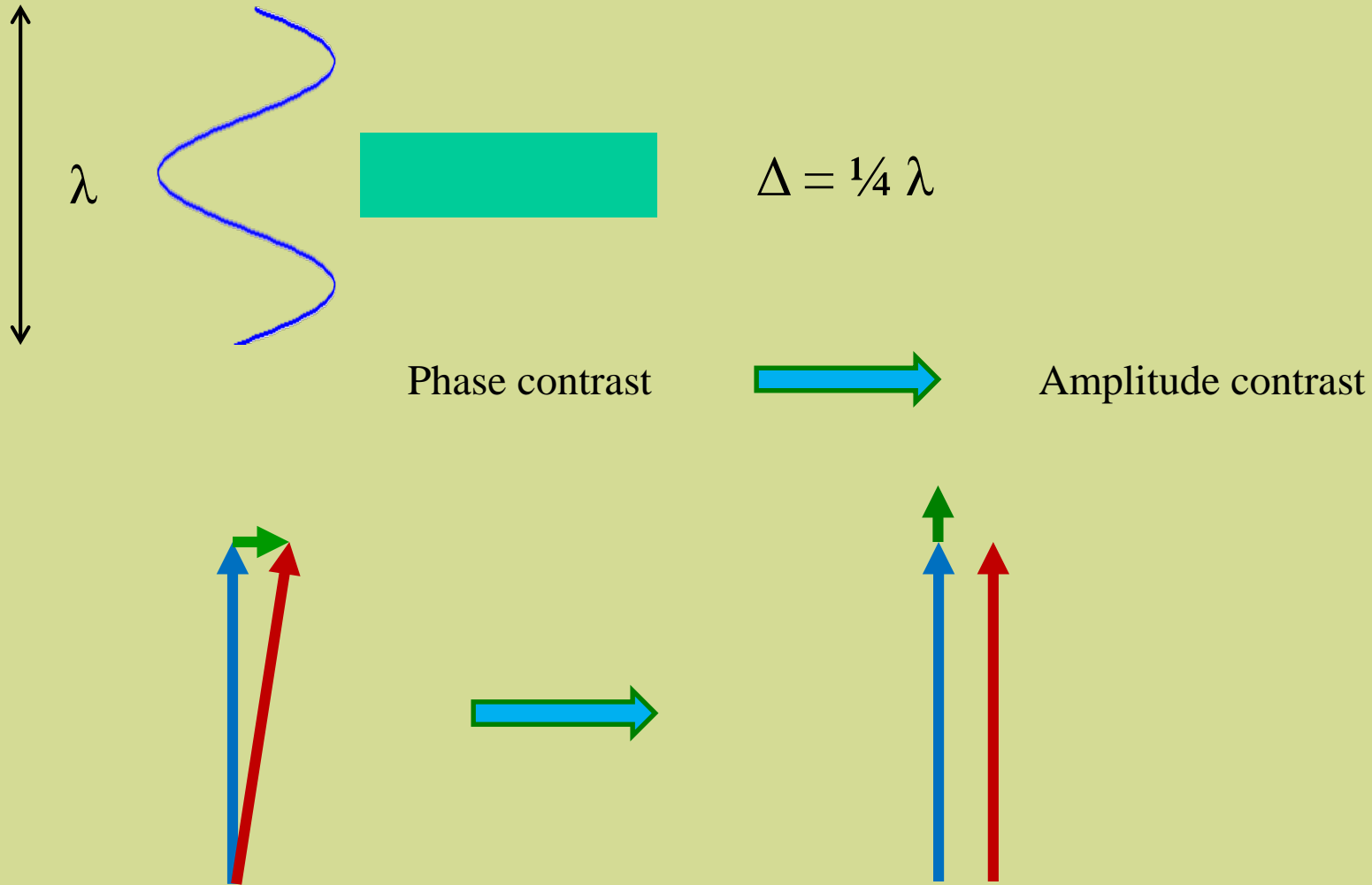
$T(x,y) = A \exp[i\phi(x,y)]$ ;  $A$  varies, linear contrast transfer; even small variations are visible

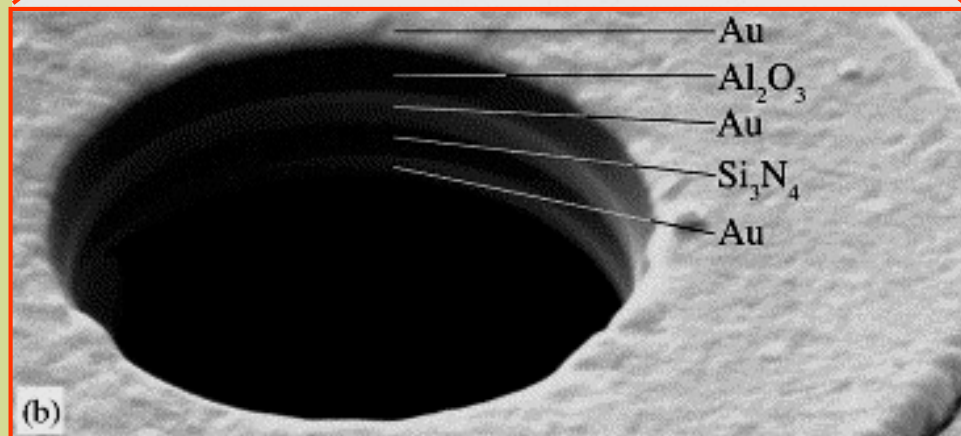
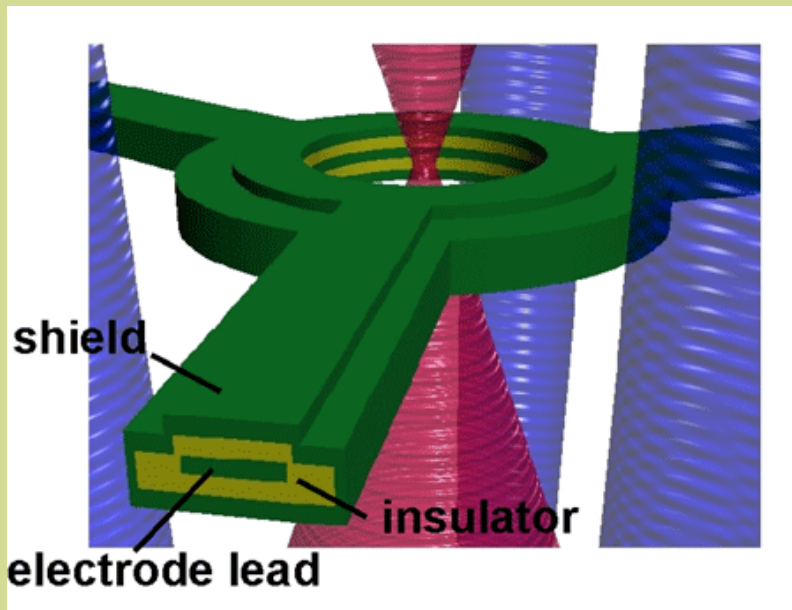
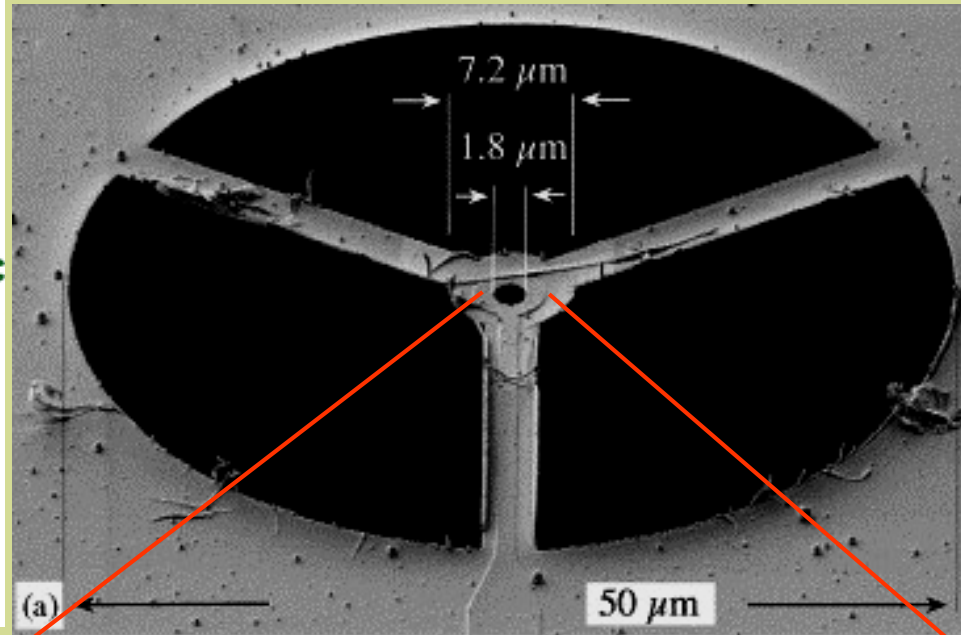
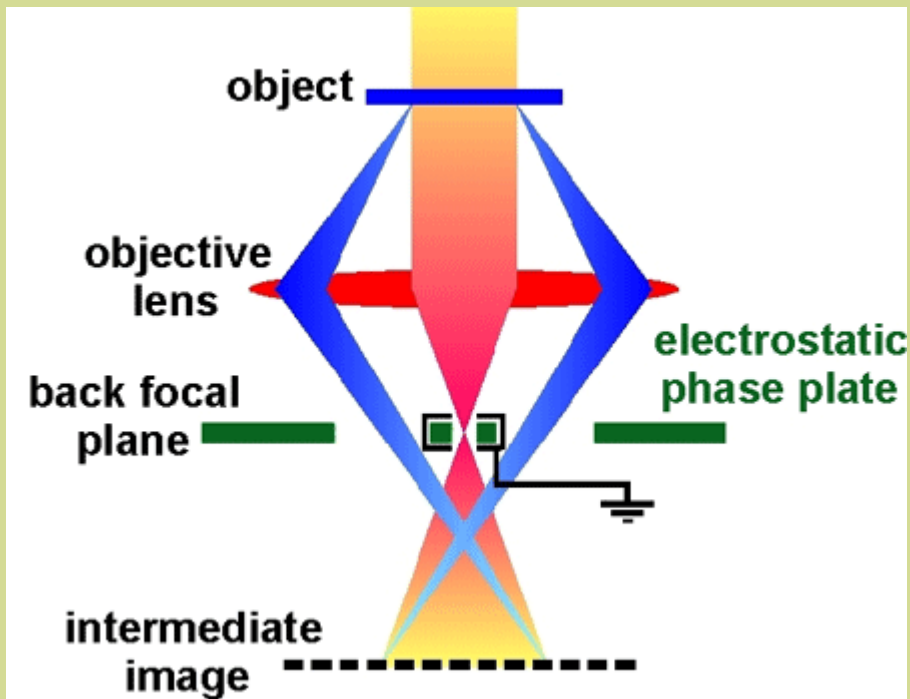
# Weak Phase <-> amplitude contrast image



# Phase plate

(phase  $\rightarrow$  amplitude contrast conversion)





Majorovits E, Barton B, Schultheiss K, Perez-Willard F, Gerthsen D, Schroder RR. Ultramicroscopy. 2007 107(2-3):213-26.

# Phase plates

K. Nagayama *Another 60 years in electron microscopy* S53

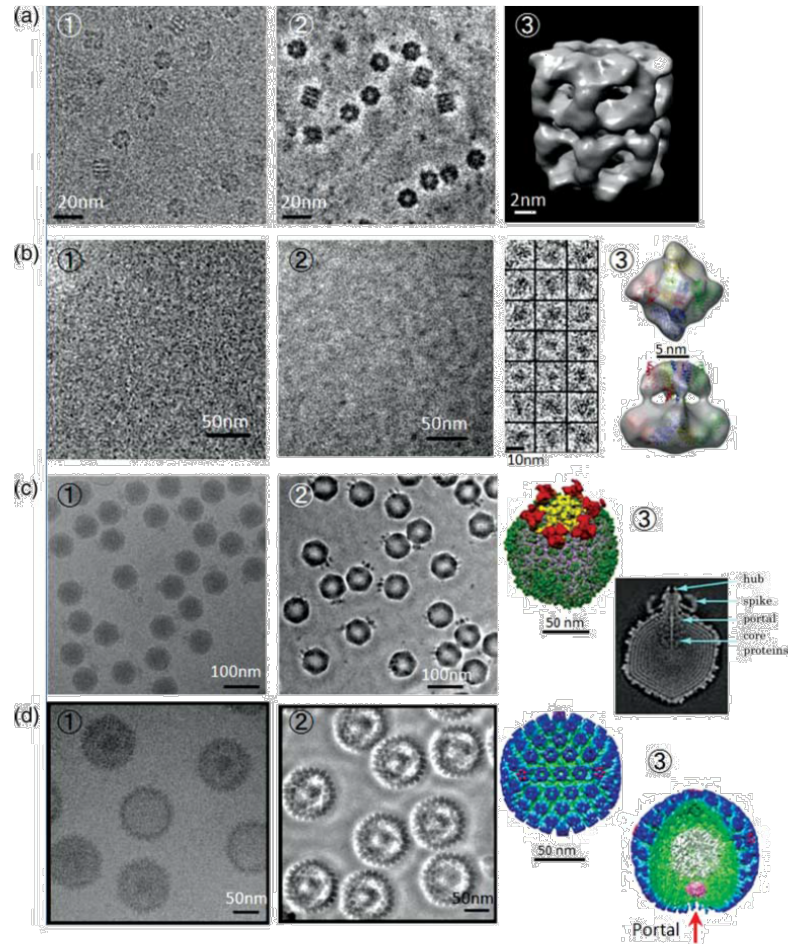
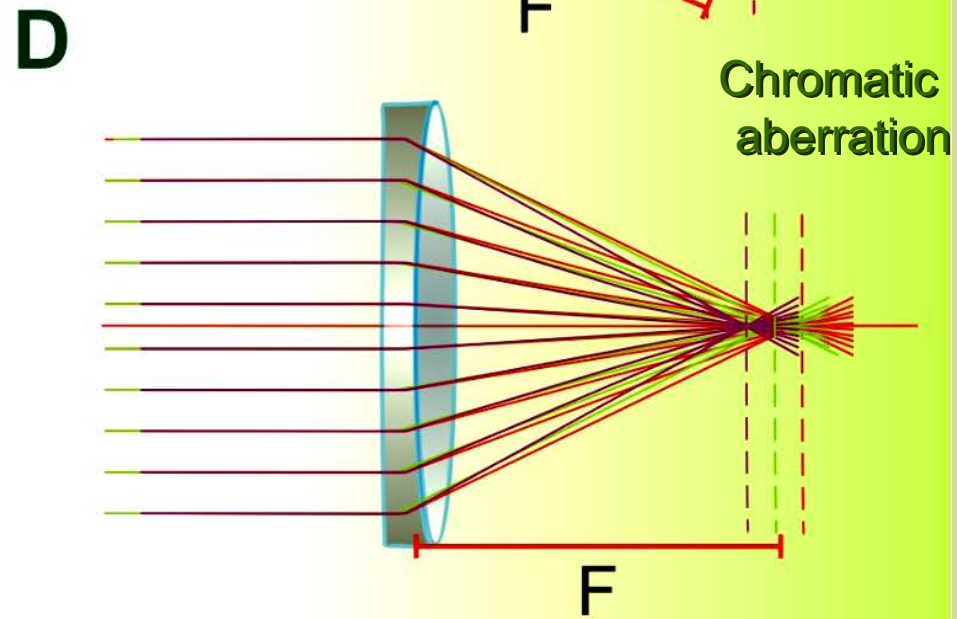
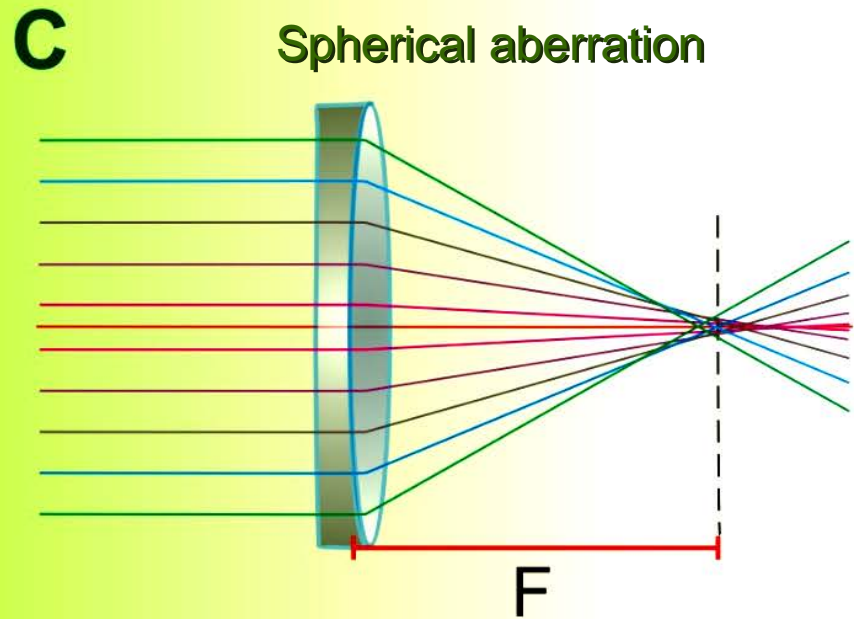
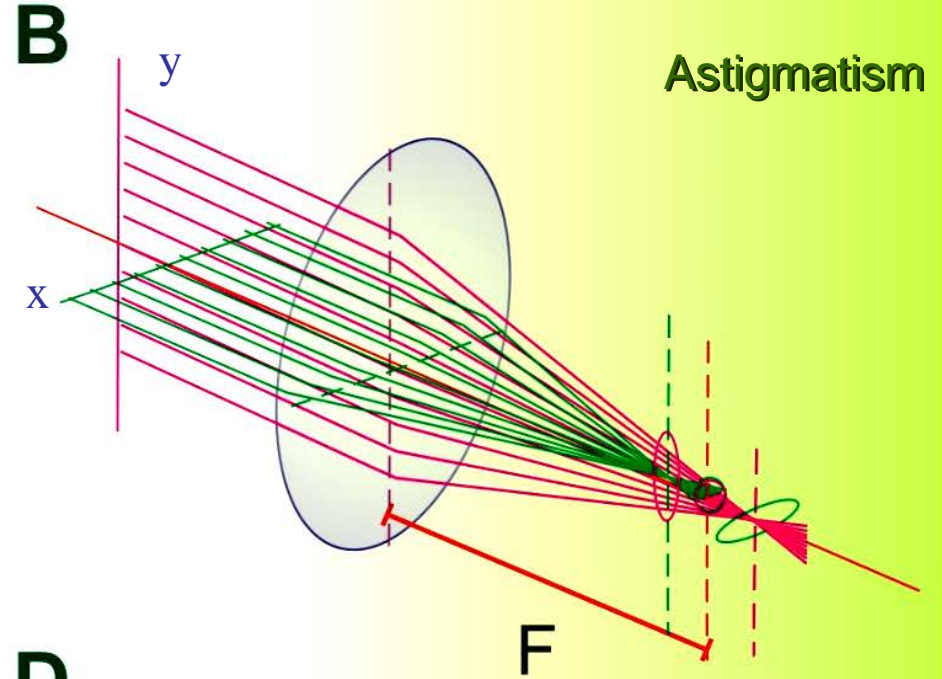
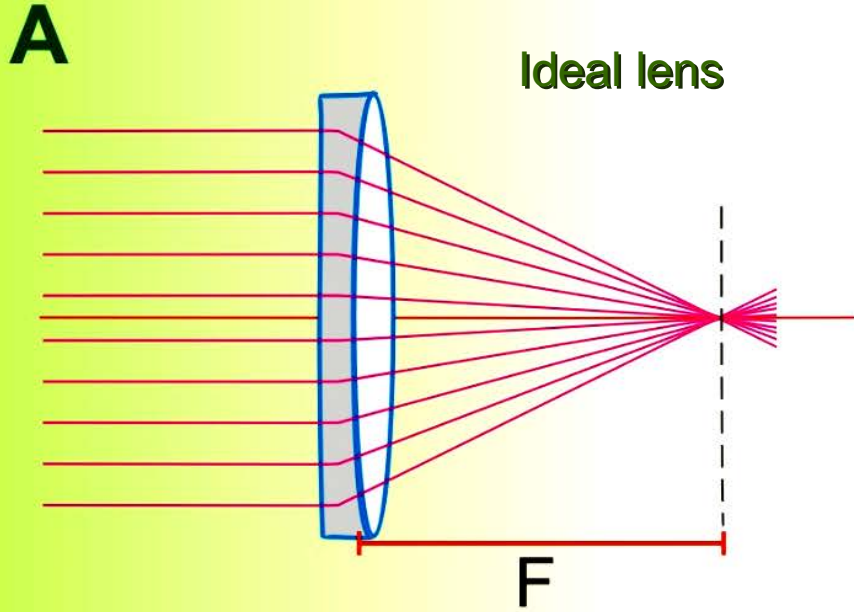


Fig. 9. Four examples of single-particle analysis based on cryo-electron microscopy. (a) A protein, GroEL (1: DPC image (300 kV), 2: ZPC image (300 kV), 3: 3D model) [from Fig. 2 of ref. 25]. (b) A membrane protein, TRPV4 (1: DPC image (300 kV), 2: ZPC image (300 kV), 3: 3D model) [from Figs. 3 and 6 of ref. 26]. (c) A bacteriophage, epsilon 15 (1: DPC image (200 kV), 2: ZPC image (200 kV), 3: 3D model) [from Figs. 2 and 3 of ref. 27]. (d) A capsid of herpes simplex virus type 1 (1: DPC image (200 kV), 2: ZPC image (200 kV), 3: 3D model) [from Figs. 1 and 2 of ref. 29].

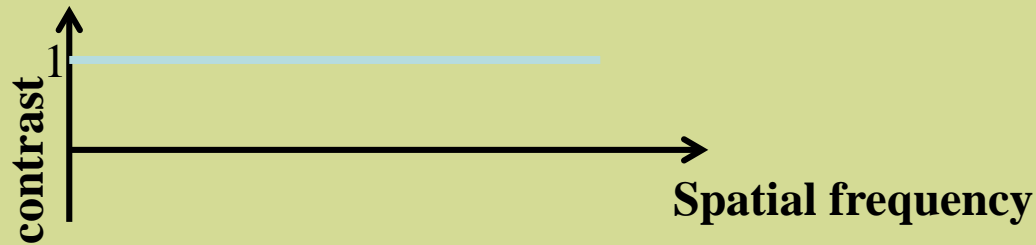


# Aberrations in optics

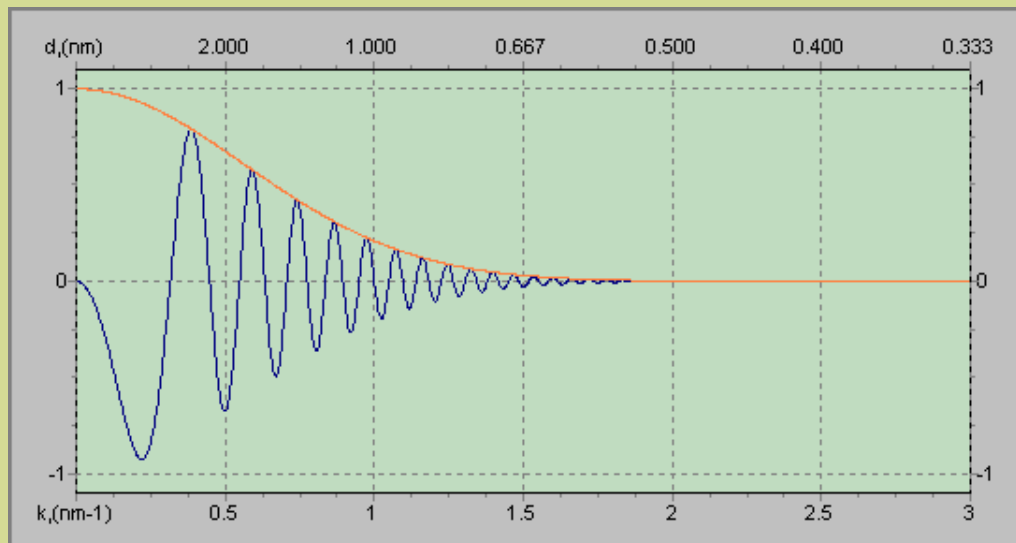


# Contrast Transfer Function (CTF) of a microscope

Ideal microscope



Real microscope



# Contrast Transfer Function (CTF) of the microscope

- $\rho_{\text{im}} = \rho_{\text{obj}} \otimes \text{PSF}$  (*real space*). PSF - Point Spread Function
  - $\text{CTF} = \mathbf{F}(\text{PSF})$
  - $\rho_{\text{im}} = \rho_{\text{obj}} \otimes \mathbf{F}^{-1}(\text{CTF})$  (*real space*).  $\mathbf{F}^{-1}(\text{CTF})$  - Point Spread Function
  - $I_{\text{im}} = I_{\text{obj}} \cdot \mathbf{H}$ , where  $\mathbf{H} = \text{CTF} \cdot \text{envelopes}$  (*reciprocal, or diffraction, or Fourier space*)
- 
- $\text{CTF}(\omega) = \sqrt{(1 - a^2)} \cdot \sin(\Gamma(\omega)) + a \cdot \cos(\Gamma(\omega))$
  - $\Gamma(\omega) \approx -1/4 C_s \omega^4 + 1/2 \Delta z \omega^2$
  - Astigmatism:  
 $\Delta z \Rightarrow 1/2 (\Delta z_{\text{max}} + \Delta z_{\text{min}} + 2(\Delta z_{\text{max}} - \Delta z_{\text{min}}) \cos(2\gamma))$ ;  
 $\gamma$  - angle from x-axis to major axis of astigmatism
  - CTF is largely restorable except for places where it is close to zero

# Point spread function

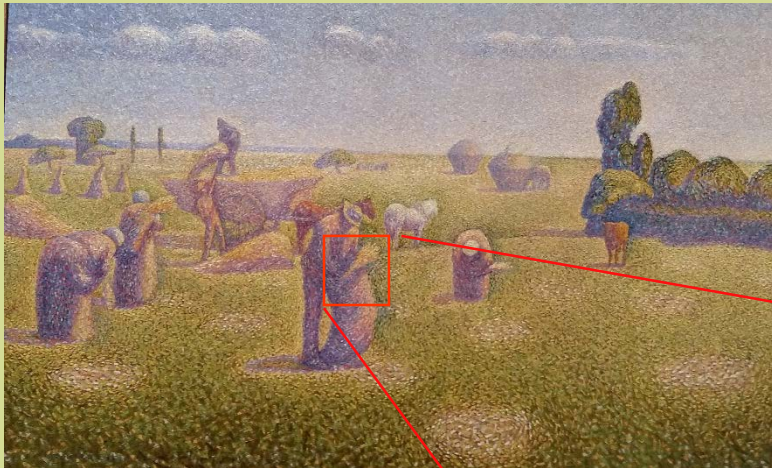


Thin pointed brush





Point spread function



Flat brush



# Convolution

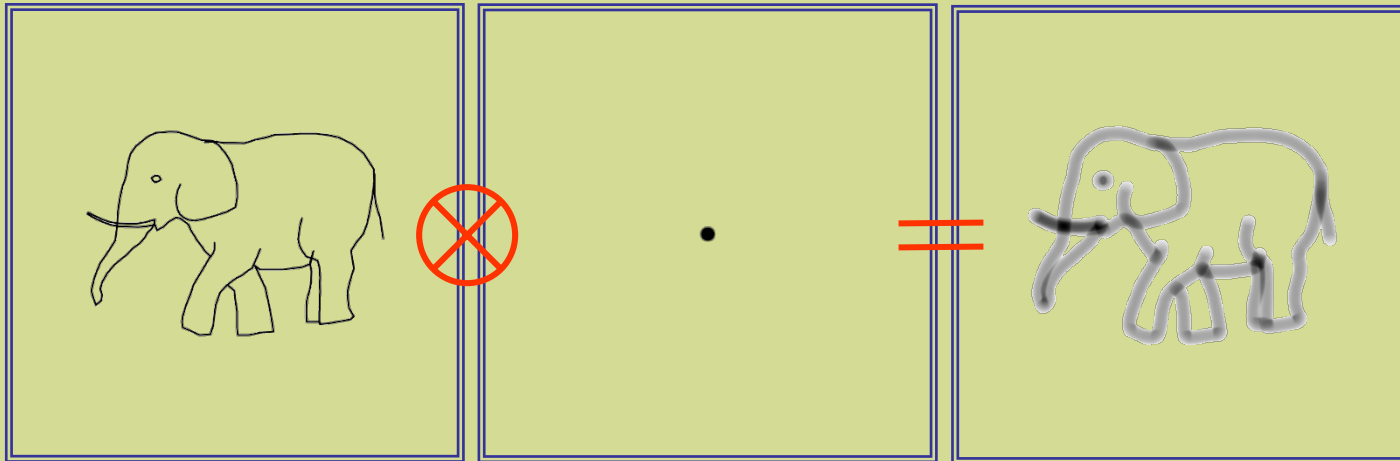
Convolution is a function describing distribution of one function by the other. It is defined as:

$$g(t) = \int g(x)h(t-x)dx = g(x) * h(x),$$

Or:

FT(J) = FT(G)\*FT(H), where FT is Fourier transform

# Point spread function and convolution



Convolution of two functions

Point Spread  
Function  
**PSF**

Image space

Real space

Density



Contrast Transfer  
Function  
**CTF**

Fourier space

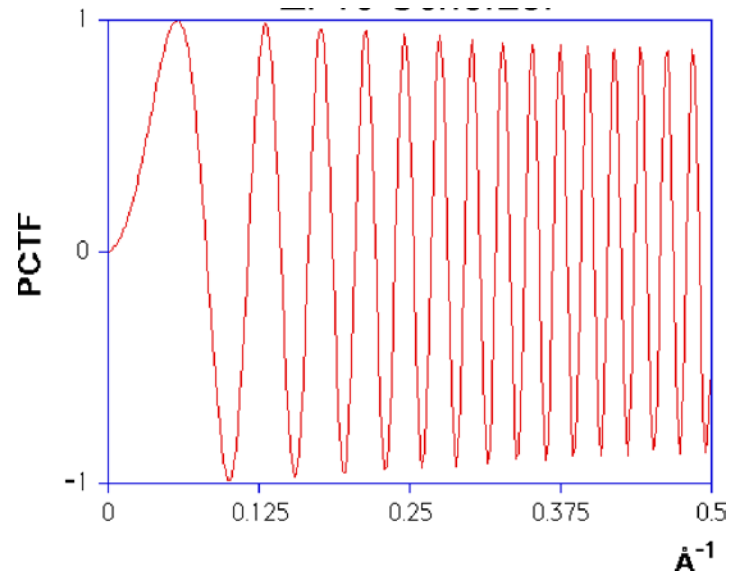
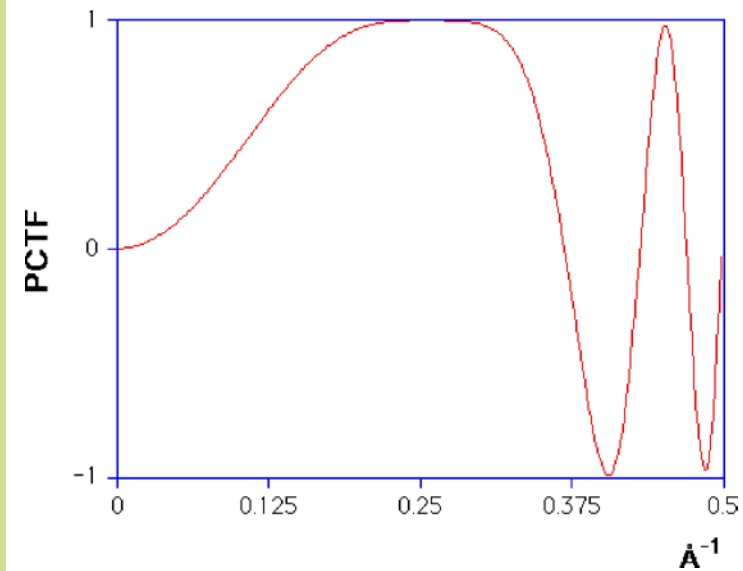
Reciprocal space

Amplitudes  
and phases



$\Delta$ : 1 Scherzer

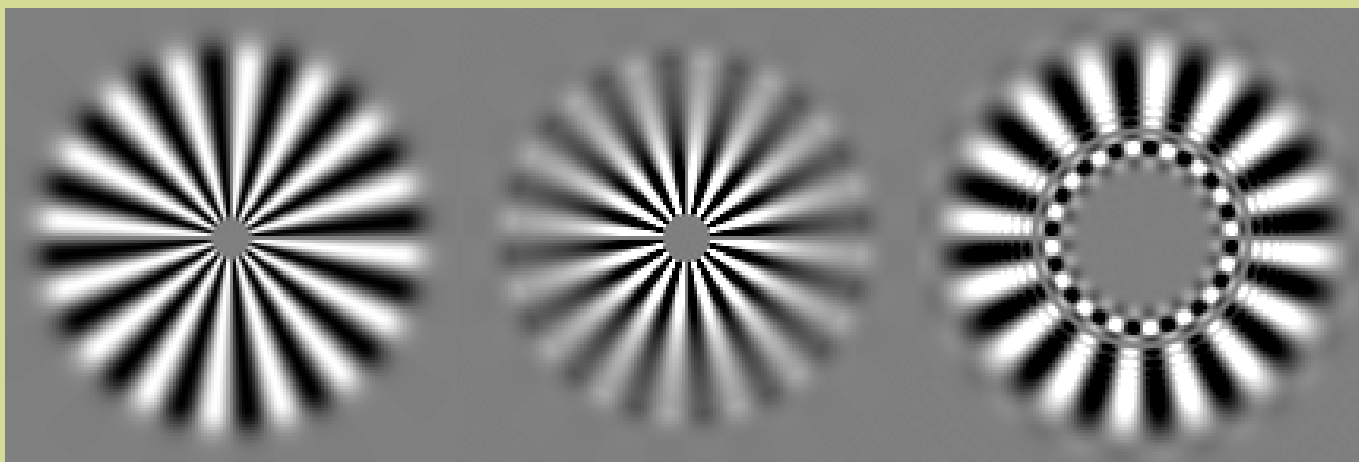
$\Delta$ : 10 Scherzer

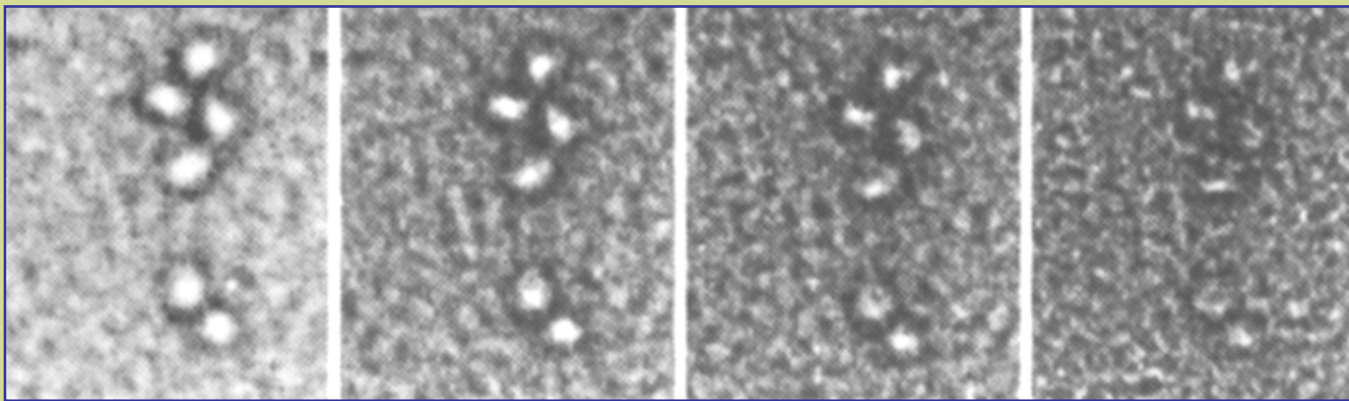


original

$\Delta$ : 1 Scherzer

$\Delta$ : 10 Scherzer



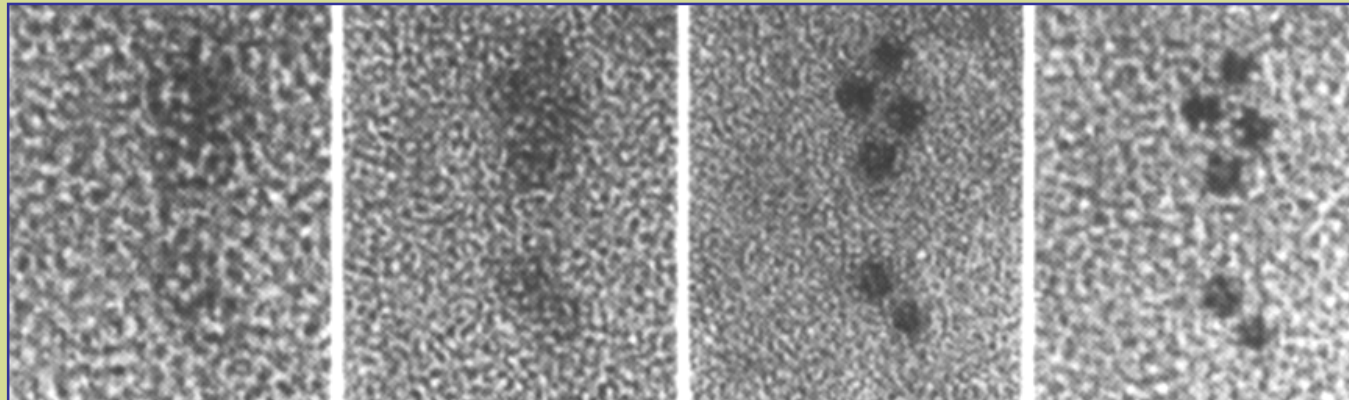


$\Delta Z = 4.8 \mu\text{m}$

$4.0 \mu\text{m}$

$3.2 \mu\text{m}$

$2.4 \mu\text{m}$

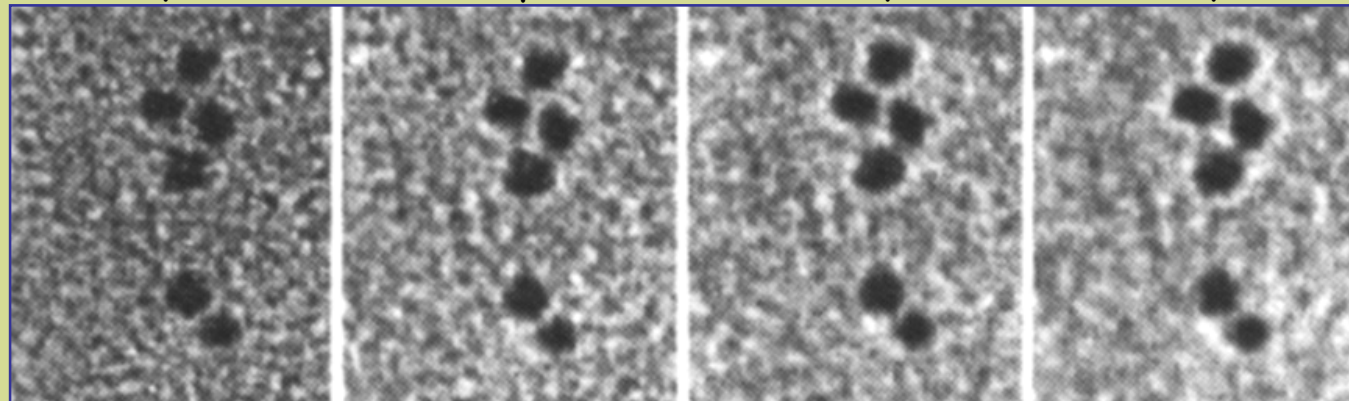


$1.6 \mu\text{m}$

$0.8 \mu\text{m}$

$\sim 0. \mu\text{m}$

$-0.8 \mu\text{m}$



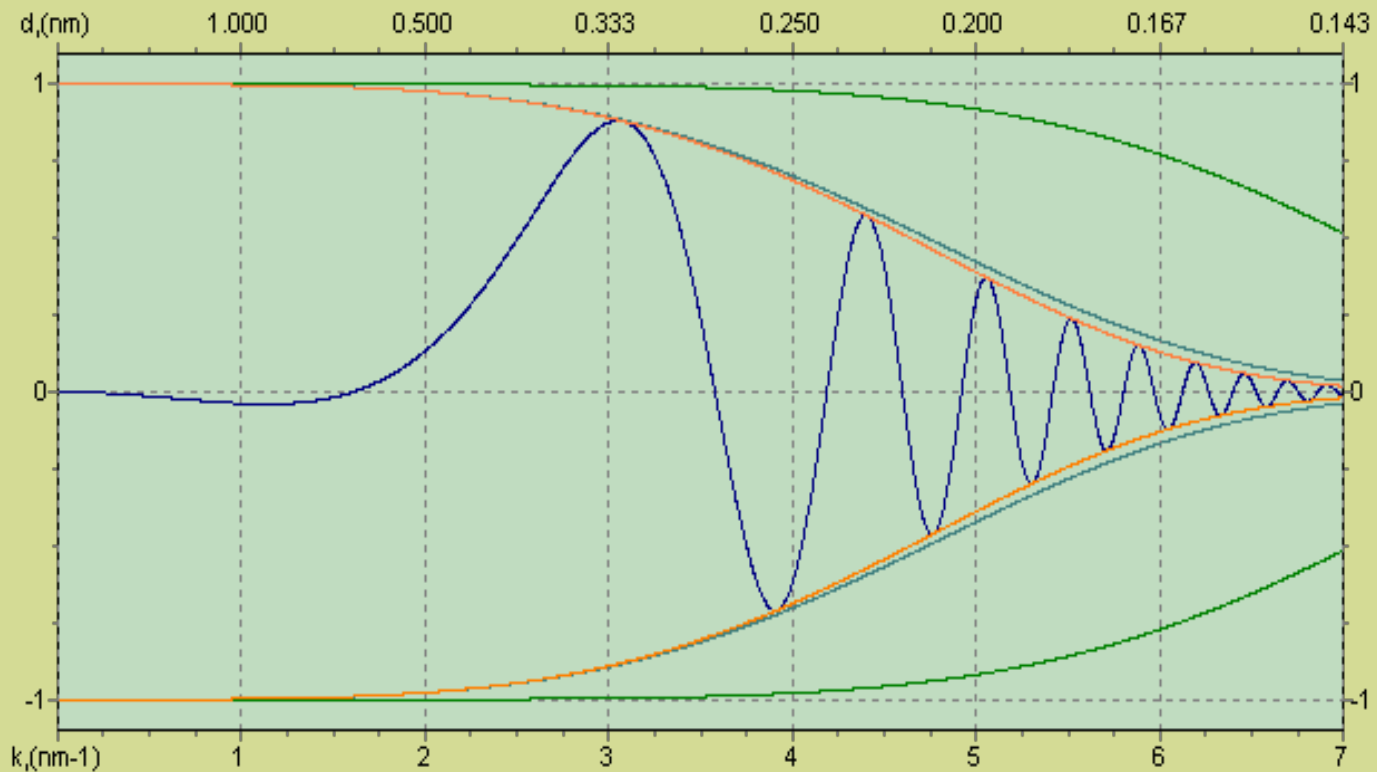
$-1.6 \mu\text{m}$

$-2.4 \mu\text{m}$

$-3.2 \mu\text{m}$

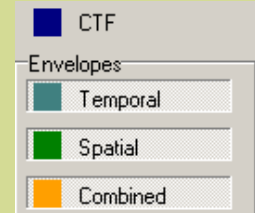
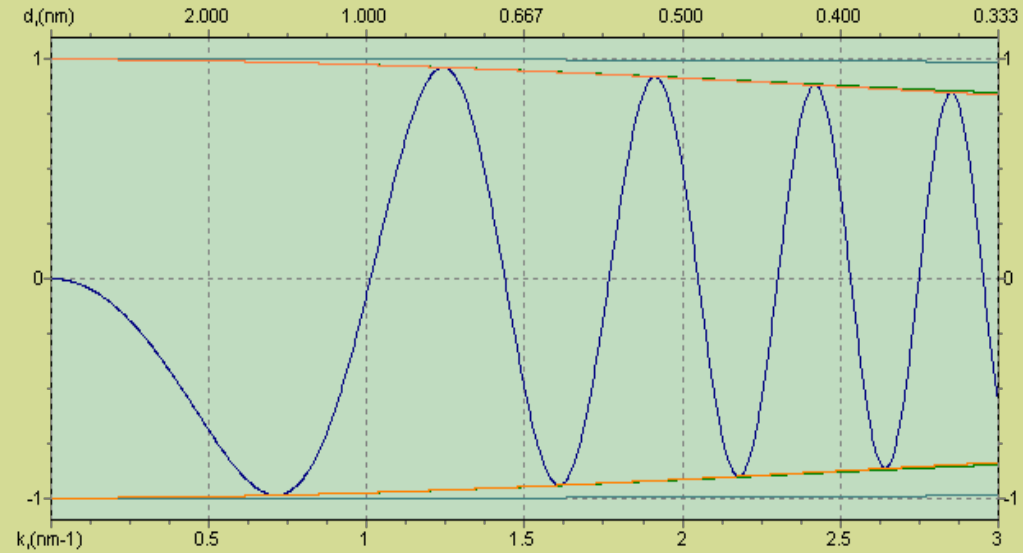
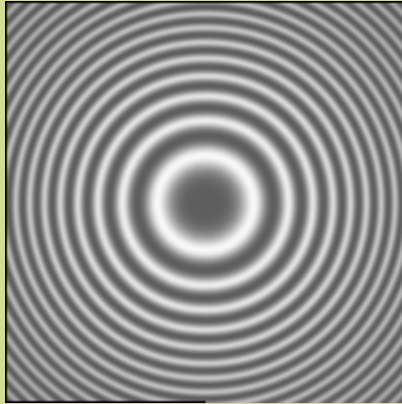
$-4.0 \mu\text{m}$

**Defocus series  
of ferritin  
molecules on a  
carbon support  
film,  $V=100\text{keV}$**



ctfexplorer

CTF (blue) and envelope functions (green and light blue)  
vs. spatial frequency



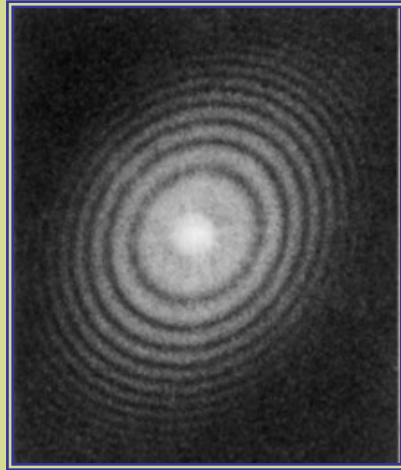
- CM 300 EM (FEI, 300 keV) image spectrum.  
500 nm defocus.

# Contrast Transfer Function (CTF) of a microscope

- Envelopes (envelope functions):
  - HT instability,
  - Lens current instability,
  - Spatial and temporal (energy spread) coherence of the primary beam.
  - Electromagnetic stray fields.
  - Vibrations.
- They all cause signal falloff at high resolution, some of them are defocus-dependent. **These are destructive defects**

# Examples of image power spectra with different defects

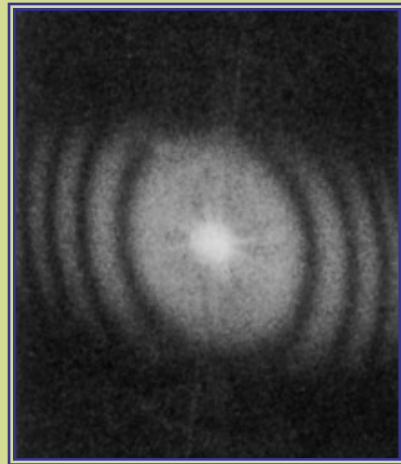
Large defocus,  
astigmatism



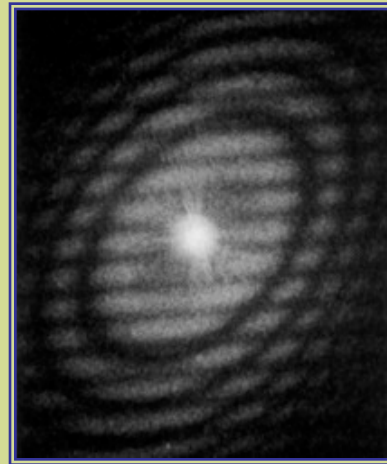
Strong  
astigmatism



Specimen drift,  
astigmatism

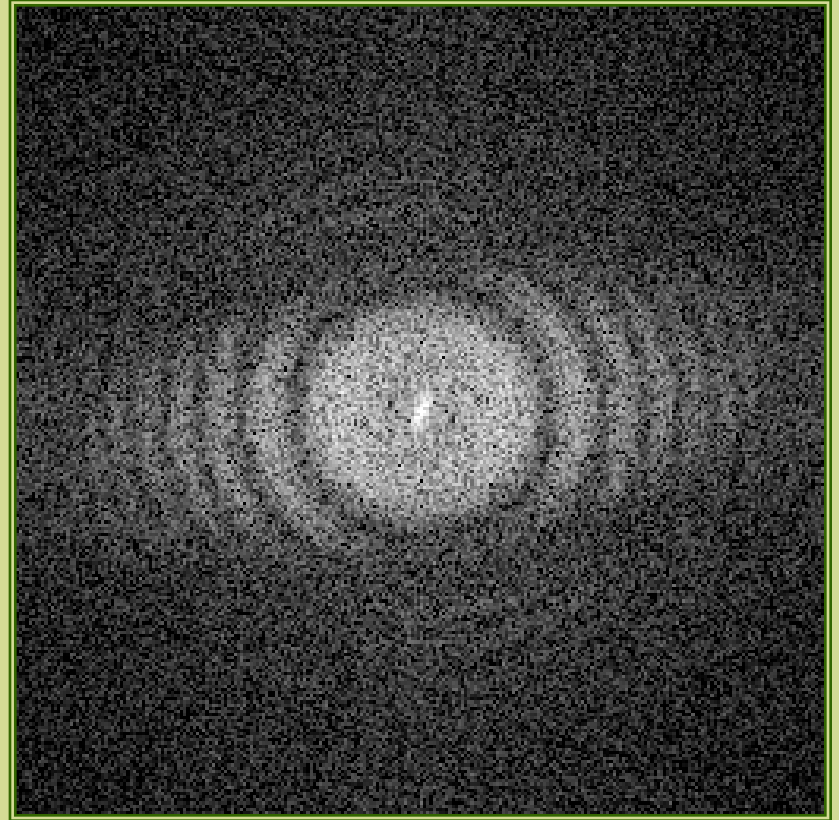
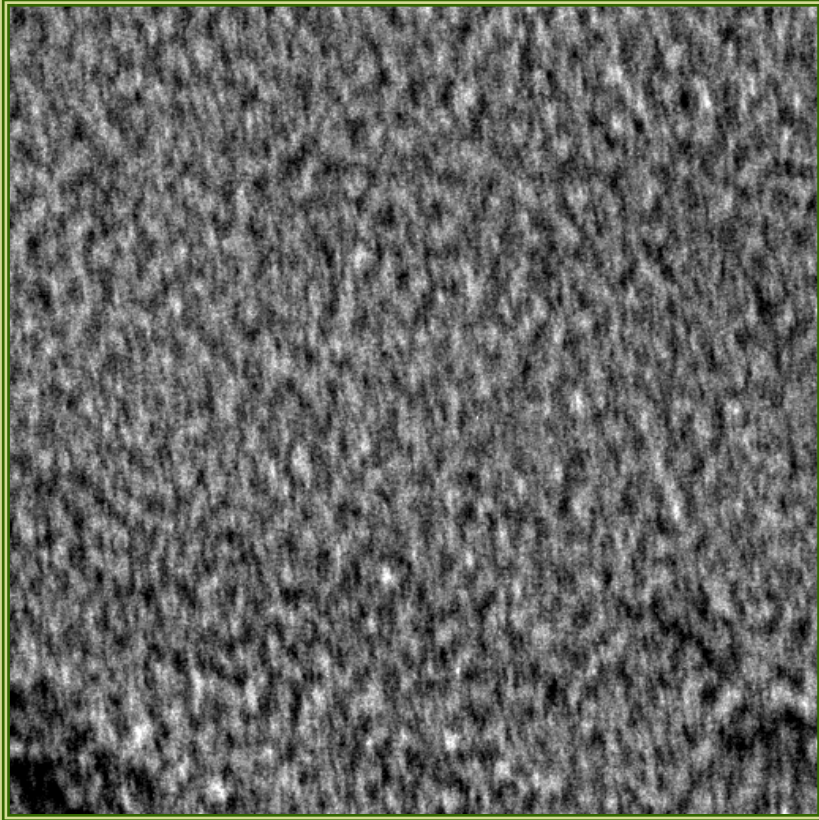


Sum of two images  
shifted relative to each  
other,  
astigmatism (Young  
fringes)





# Drift



# CTF correction; Wiener filtering

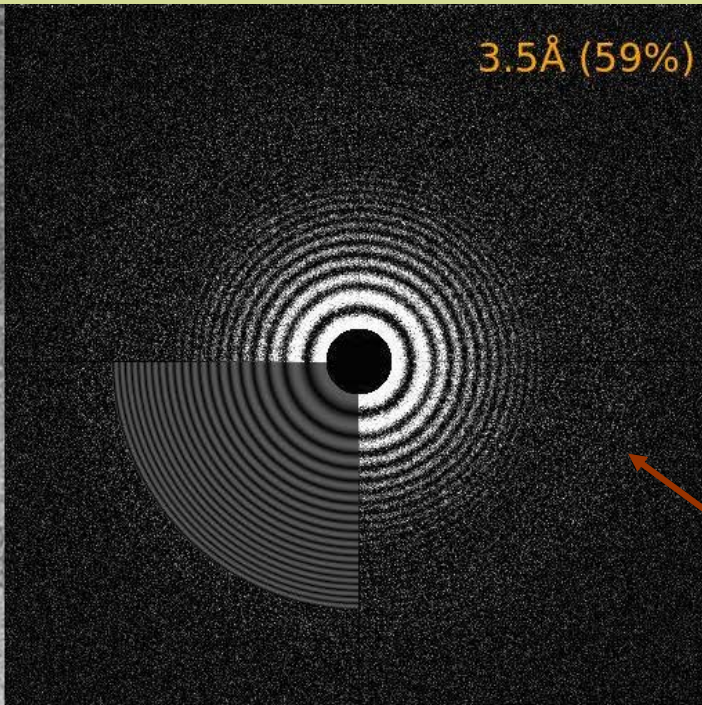
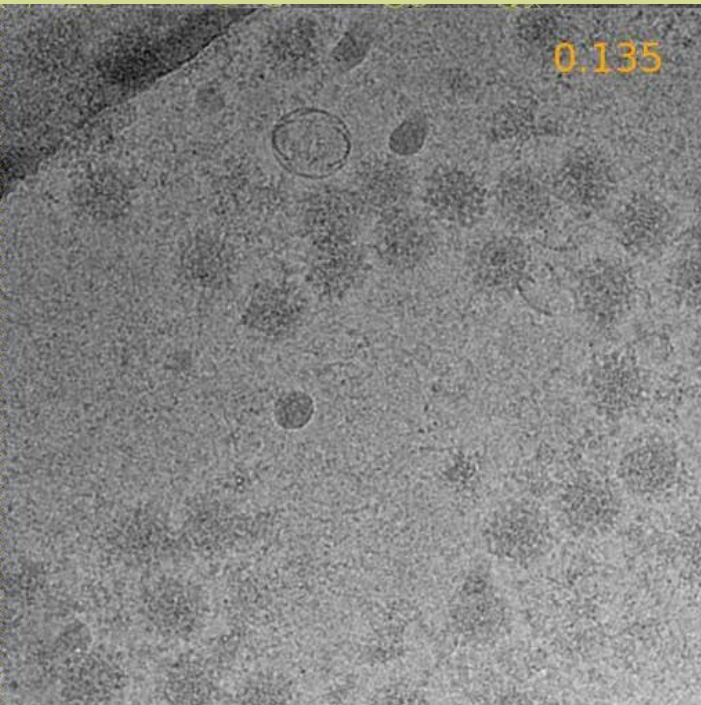
- $I_{im} = I_{obj} \cdot H$ , where  $H = \text{CTF} \cdot \text{envelopes}$

- $I_{restored} = \frac{I_{im}}{H}$ , where  $H \neq 0$  (no noise) or, better:

$$I_{restored} = \frac{I_{im} \cdot H}{H^2 + \left(\frac{N}{S}\right)^2}, \text{ where } N \text{ is noise, and } S \text{ is signal}$$

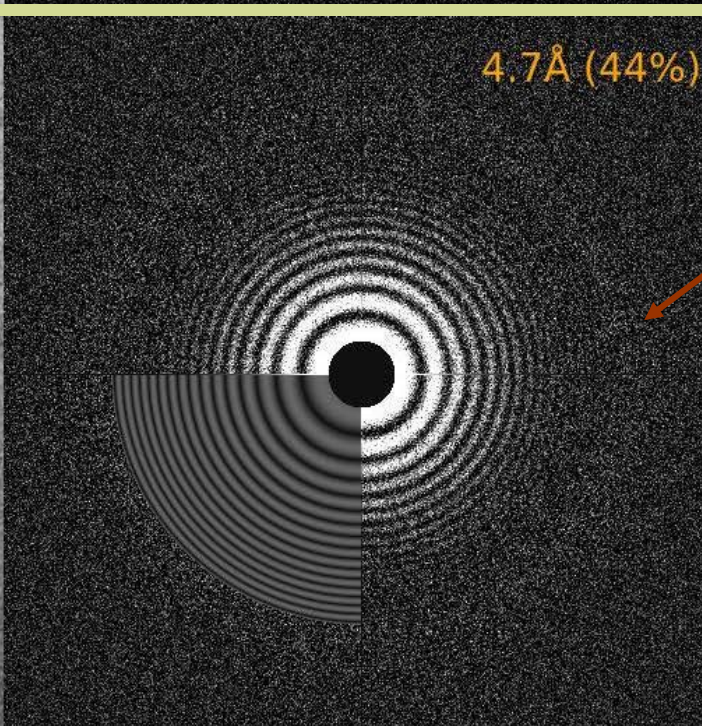
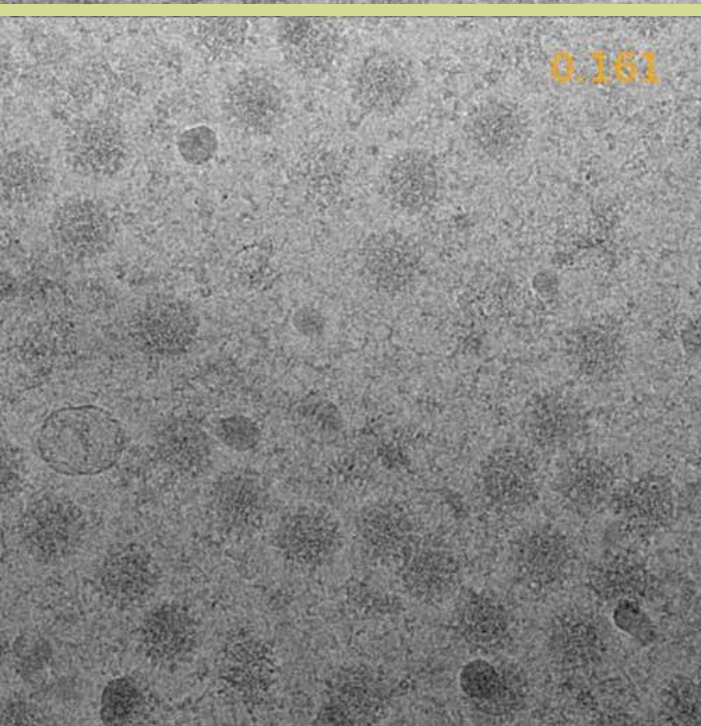
- If signal is strong, then  $I_{restored} = I_{im}/H$ ; but
- If signal is weak then  $I_{restored} \approx 0$
- The latter formula is called “Wiener filter”, an optimal filter



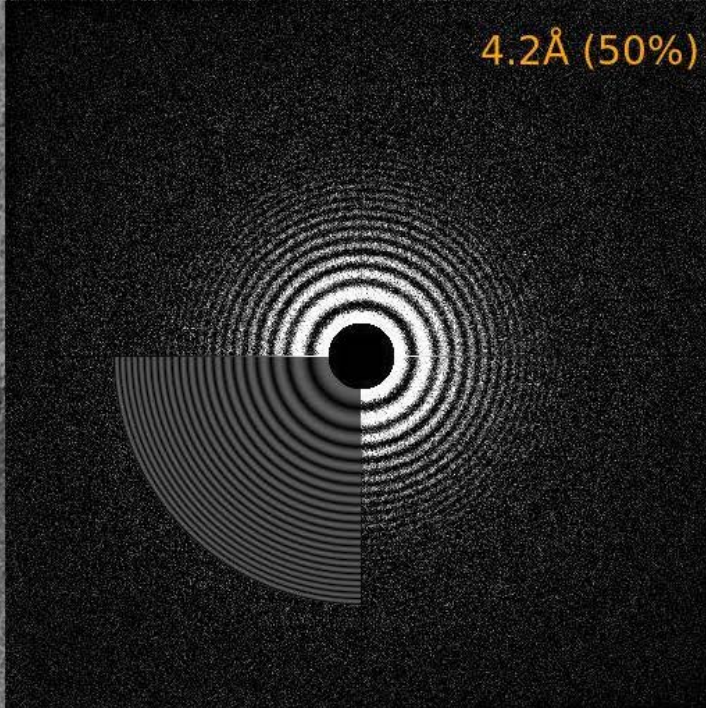
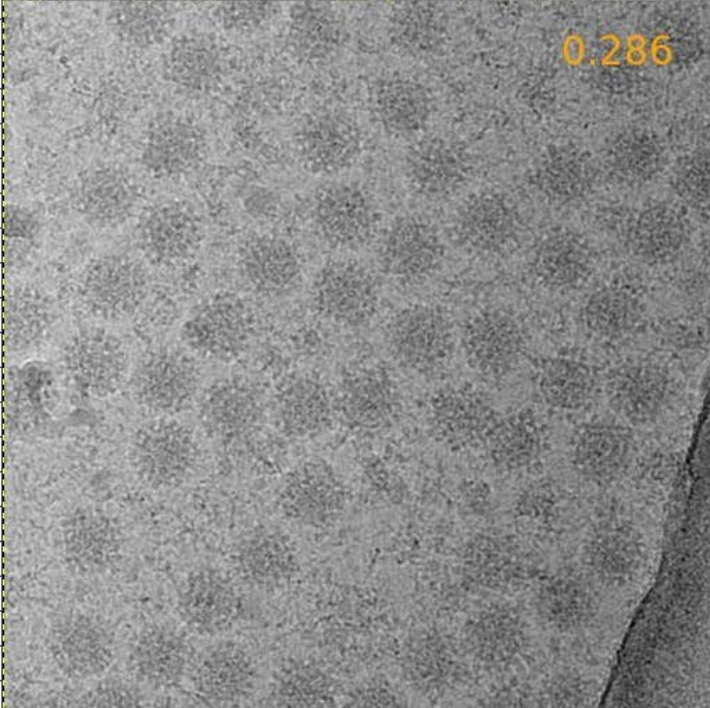


CTFFIND,  
frame alignment

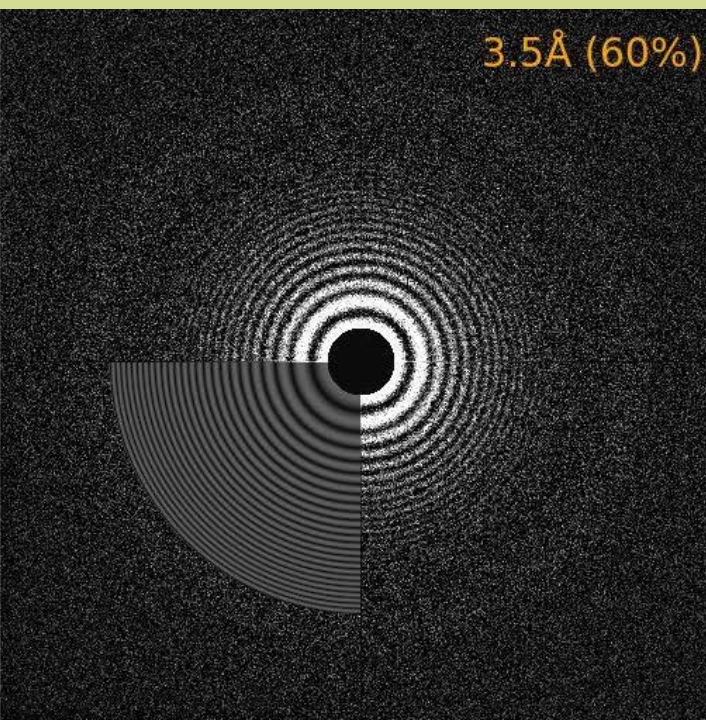
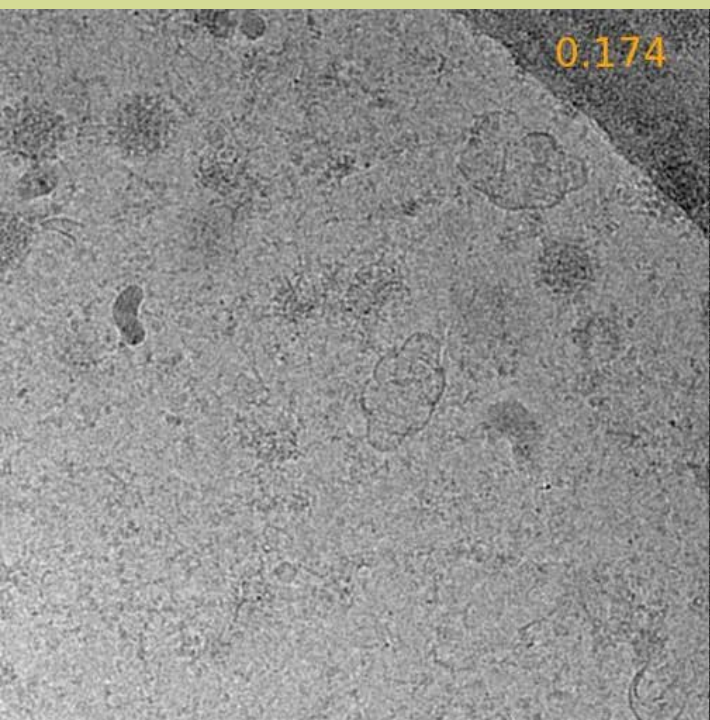
Water scattering ring





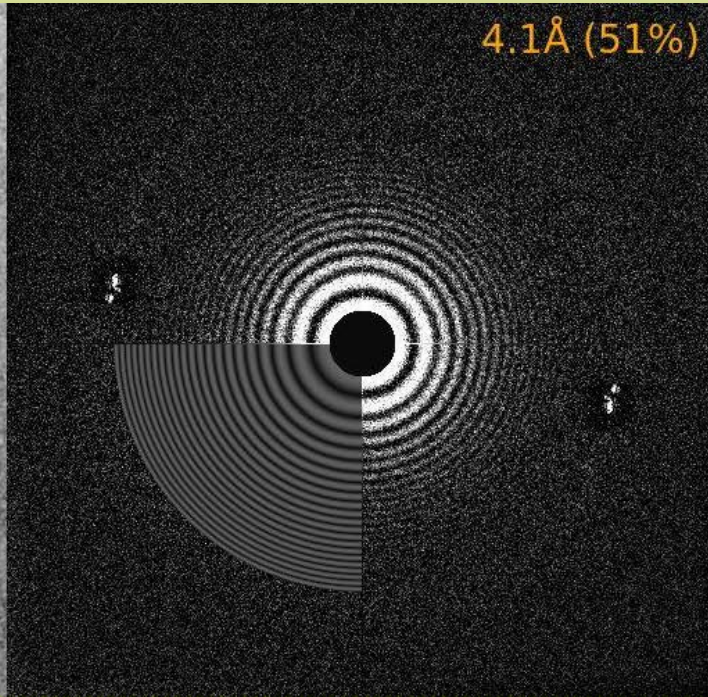
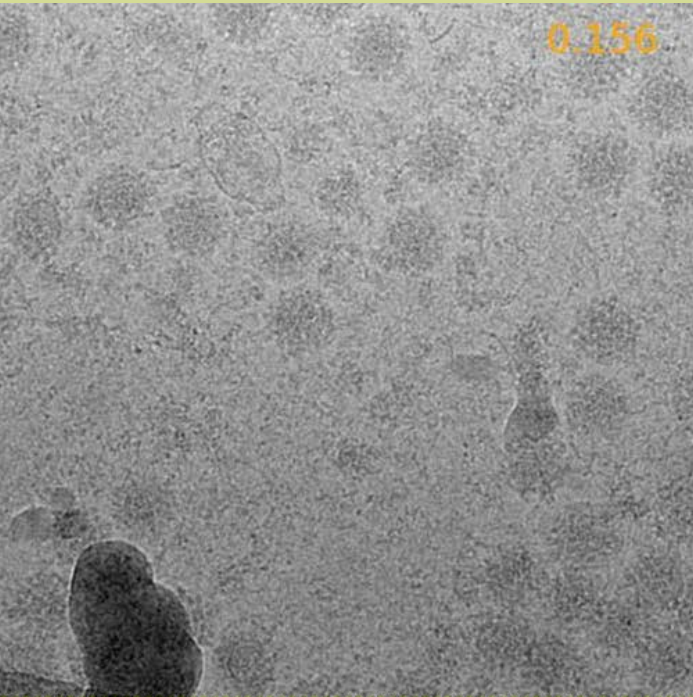


Good image

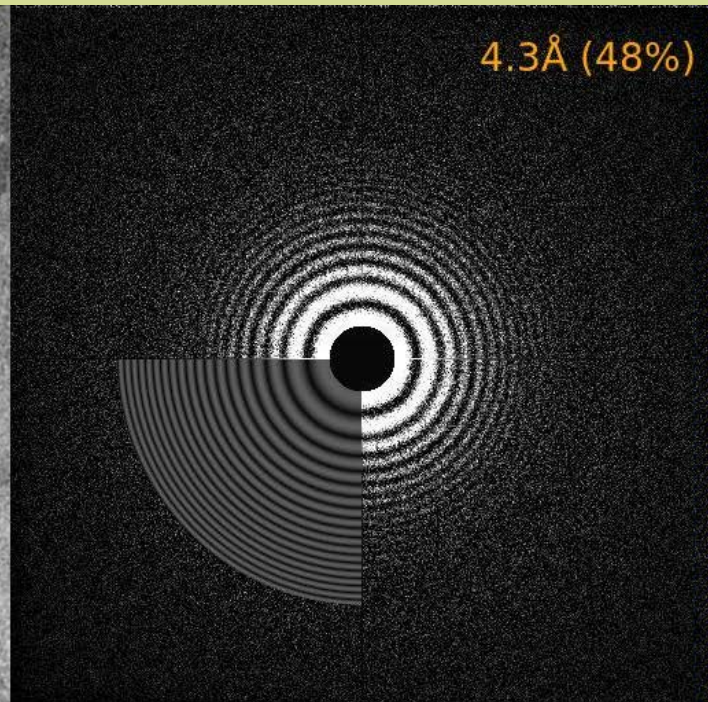
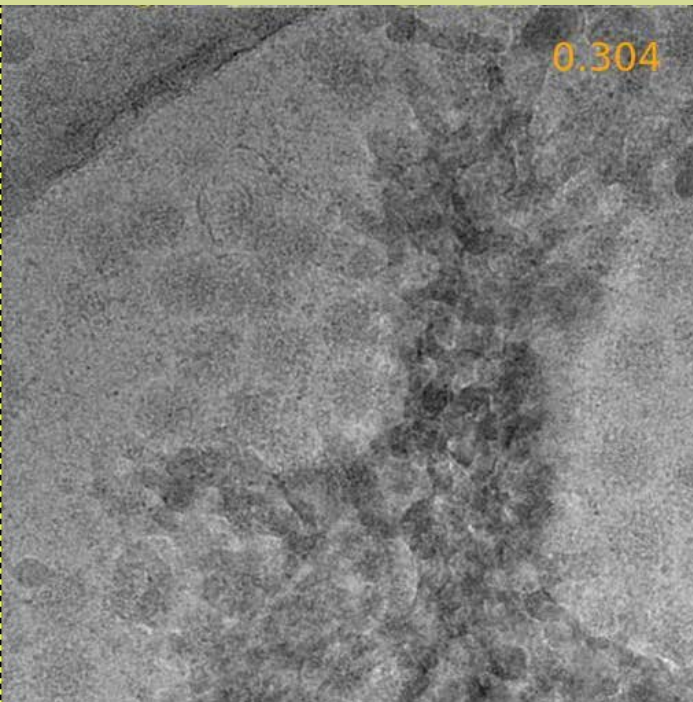


Good image,  
No sample





Crystalline ice present



Heavy contamination

## References

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